MARTINGALE SELECTION THEOREM FOR A STOCHASTIC SEQUENCE WITH RELATIVELY OPEN CONVEX VALUES

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Abstract. For a set-valued stochastic sequence $(G_n)_{n=0}^N$ with relatively open convex values $G_n(\omega)$ we give a criterion for the existence of an adapted sequence $(x_n)_{n=0}^N$ of selectors, admitting an equivalent martingale measure. Mentioned criterion is expressed in terms of supports of the regular conditional upper distributions of the elements G_n. This result is a refinement of the main result of author's previous paper (Teor. Veroyatnost. i Primen., 2005, 50:3, 480–500), where the sets $G_n(\omega)$ were assumed to be open and where were asked if the openness condition can be relaxed.

Introduction

Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space endowed with the filtration $(\mathcal{F}_n)_{n=0}^N$. Consider a sequence of $\mathcal{F}_n$-measurable set-valued maps $\Omega \mapsto G_n(\omega) \subset \mathbb{R}^d$, $n = 0, \dots, N$ with the nonempty relatively open convex values $G_n(\omega)$. In this paper we give a criterion for the existence of a pair, consisting of an adapted single-valued stochastic process $x = (x_n)_{n=1}^N$, $x_n(\omega) \in G_n(\omega)$ and a probability measure $\mathbf{Q}$ equivalent to $\mathbf{P}$ such that x is a martingale under $\mathbf{Q}$. Following [1], we say that *the martingale selection problem* (m.s.p.) is solvable if such a pair $(x, \mathbf{Q})$ exists.

This problem is motivated by some questions of arbitrage theory. In particular, if the mappings G_n are single-valued, then we obtain the well-known problem concerning the existence of an equivalent martingale measure for a given stochastic process $G_n = x_n$. In this case the solvability of the m.s.p. is equivalent to the absence of arbitrage in the market, where the discounted asset price process is described by x [2–5]. It is shown in [4] that an equivalent martingale measure for x exists iff the convex hulls of the supports of $x_n - x_{n-1}$ regular conditional distributions with respect to $\mathcal{F}_{n-1}$ contain the origin as a point of relative interior [4, Theorem 3, condition (g)]. The aim of the present paper is to refine this result.

In the framework of market models with transaction costs [6–8] the role of equivalent martingale measures is played by strictly consistent price processes. This name is assigned to $\mathbf{P}$-martingales a.s. taking values in the relative interior of the random cones K^*, conjugate to the solvency cones K. Using the invariance of K under multiplication, it is easy to show (see [1, Introduction]) that the existence of a strictly consistent price process is

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equivalent to the solvability of the m.s.p. for the sequence $(\operatorname{ri} K_n^*)_{n=0}^N$ of relative interiors of K_n^*.

In the paper [1] there was obtained a criterion of the solvability of the m.s.p. under the assumption that the sets $G_n(\omega)$ are open. This result is not completely satisfactory since, for instance, it does not include the case of single-valued G_n and it does not allow the cones K_n^* to have the empty interior. The last limitation means that the "efficient friction" condition must be satisfied (according to the terminology of [6]).

In the present paper we refine the main result of [1] (see Theorem 1). Moreover, the proof given below, as compared to [1], is considerably simplified.

2. Preliminaries

Consider a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ and a σ-algebra $\mathcal{H} \subset \mathcal{F}$. In the sequel we assume that all σ-algebras are complete with respect to $\mathbf{P}$ (i.e. they contain all the subsets of their $\mathbf{P}$-negligible sets). Denote by $\operatorname{cl} A$, $\operatorname{ri} A$, $\operatorname{conv} A$ the closure, the relative interior, and the convex hull of a subset A of a finite-dimensional space. Let $\mathcal{B} = \mathcal{B}(\mathbb{R}^d)$ be the Borel σ-algebra of $\mathbb{R}^d$.

A set-valued map F, assigning some set $F(\omega) \subset \mathbb{R}^d$ to each $\omega \in \Omega$, is called $\mathcal{H}$-measurable if $\{\omega : F(\omega) \cap V \neq \emptyset\} \in \mathcal{H}$ for any open set $V \subset \mathbb{R}^d$. The graph and the domain of F are defined by

$$\operatorname{gr} F = \{(\omega, x) : x \in F(\omega)\}, \quad \operatorname{dom} F = \{\omega : F(\omega) \neq \emptyset\}.$$

If $\operatorname{gr} F \in \mathcal{H} \otimes \mathcal{B}$, then the mapping F is $\mathcal{H}$-measurable [9, Corollary II.1.34].

The function $f : \Omega \mapsto \mathbb{R}^d$ is called a *selector* of a set-valued map F if $f(\omega) \in F(\omega)$ for all $\omega \in \operatorname{dom} F$. Denote by $\mathcal{S}(F, \mathcal{H})$ the set of $\mathcal{H}$-measurable selectors of F. Note, that if the set-valued map F is $\mathcal{H}$-measurable, then the mapping

$$F_* = F I_{\operatorname{dom} F} + \mathbb{R}^d I_{\Omega \setminus \operatorname{dom} F} \tag{1}$$

is also $\mathcal{H}$-measurable and $\mathcal{S}(F, \mathcal{H}) = \mathcal{S}(F_*, \mathcal{H})$. Here $I_A(\omega) = 1$, $\omega \in A$; $I_A(\omega) = 0$, $\omega \notin A$.

The countable family $\{f_i\}_{i=1}^\infty$ of an $\mathcal{H}$-measurable selectors is called (an $\mathcal{H}$-measurable) *Castaing representation* for F, if the sets $\{f_i(\omega)\}_{i=1}^\infty$ are dense in $F(\omega)$ for all $\omega \in \operatorname{dom} F$. The set-valued map F with nonempty closed values is $\mathcal{H}$-measurable iff it admits an $\mathcal{H}$-measurable Castaing representation [9, Proposition II.2.3].

An element $f \in \mathcal{S}(\operatorname{conv} F, \mathcal{H})$ is said to have an $\mathcal{H}$-measurable *Caratheodory representation*, if there are some elements $g_k \in S(F, \mathcal{H})$, $k = 1, \dots, d+1$ and $\mathcal{H}$-measurable functions

$$\alpha_k \geq 0, \ k = 1, \dots, d+1; \ \sum_{k=1}^{d+1} \alpha_k = 1 \tag{2}$$

such that $f = \sum_{k=1}^{d+1} \alpha_k g_k$ a.s. Under the assumption $\operatorname{gr} F \in \mathcal{H} \otimes \mathcal{B}$ any element $f \in \mathcal{S}(\operatorname{conv} F, \mathcal{H})$ has an $\mathcal{H}$-measurable Caratheodory representation [10, Theorem 8.2(iii)].

Denote by $\mathrm{CL} = \mathrm{CL}(\mathbb{R}^d)$ the family of nonempty closed subsets of $\mathbb{R}^d$ and let $\mathcal{E}(\mathrm{CL})$ be the *Effros σ-algebra*, generated by the sets of the form

$$A_V = \{D \in \mathrm{CL} : D \cap V \neq \emptyset\},$$

where V is an open subset of $\mathbb{R}^d$.

Suppose F is an $\mathcal{F}$-measurable set-valued map with nonempty closed values. It follows directly from the definitions that the corresponding single-valued map $F : (\Omega, \mathcal{F}) \mapsto (\mathrm{CL}, \mathcal{E}(\mathrm{CL}))$ is measurable. The measurable space $(\mathrm{CL}, \mathcal{E}(\mathrm{CL}))$ is a Borel space ([11, Theorem 3.3.10]). Consequently, the map F, considered as a random element taking values in $(\mathrm{CL}, \mathcal{E}(\mathrm{CL}))$, has the regular conditional distribution with respect to $\mathcal{H}$ [12, Chapter II, §7, Theorem 5].

Thus, there exists a function $\mathbf{P}^* : \Omega \times \mathcal{E}(\mathrm{CL}) \mapsto [0, 1]$ with the following properties:

(i) for every ω the function $C \mapsto \mathbf{P}^*(\omega, C)$ is a probability measure on $\mathcal{E}(\mathrm{CL})$;

(ii) for every $C \in \mathcal{E}(\mathrm{CL})$ the function $\omega \mapsto \mathbf{P}^*(\omega, C)$ a.s. coincides with the conditional probability $\mathbf{P}(\{F \in C\}|\mathcal{H})(\omega)$.

Following [1], we define the *regular conditional upper distribution* of the mapping F with respect to $\mathcal{H}$ by the formula $\mu_{F,\mathcal{H}}(\omega, V) = \mathbf{P}^*(\omega, A_V)$ for any open subset $V \subset \mathbb{R}^d$. The set

$$\mathcal{K}(F, \mathcal{H}; \omega) = \big\{y \in \mathbb{R}^d : \mu_{F,\mathcal{H}}(\omega, \{y' : |y' - y| < \varepsilon\}) > 0 \text{ for all } \varepsilon > 0\big\}$$

is called the *support* of $\mu_{F,\mathcal{H}}(\omega, \cdot)$ [1]. Note, that if F is a single-valued map, then $\mu_{F,\mathcal{H}}$ is its regular conditional distribution with respect to $\mathcal{H}$ and $\mathcal{K}(F, \mathcal{H})$ is the support of the measure $\mu_{F,\mathcal{H}}$.

The set-valued map $\omega \mapsto \mathcal{K}(F, \mathcal{H}; \omega)$ has nonempty closed values and is $\mathcal{H}$-measurable [1, Proposition 4(a)]. Let $\{f_i\}_{i=1}^\infty$ be an $\mathcal{F}$-measurable Castaing representation for F. Then the following equality holds true (see [1, Lemma 1]):

$$\mathcal{K}(F, \mathcal{H}) = \mathrm{cl}\left(\bigcup_{i=1}^\infty \mathcal{K}(f_i, \mathcal{H})\right) \text{ a.s.} \tag{3}$$

If the values of F are empty on a $\mathbf{P}$-null set, then we put $\mathcal{K}(F, \mathcal{H}) = \mathcal{K}(F_*, \mathcal{H})$, where F_* is defined by (1). Evidently, equaility (3) still holds true in this case.

Provided $F(\omega) = \emptyset$ on a set of positive measure, we put $\mathcal{K}(F, \mathcal{H}) = \emptyset$ for all ω.

3. Main result

Suppose $\Omega \mapsto G_n(\omega) \subset \mathbb{R}^d$, $n = 0, 1, \ldots, N$ is a sequence of $\mathcal{F}_n$-measurable set-valued maps with nonempty relatively open convex values $G_n(\omega)$. Define the sequence $(W_n)_{n=0}^N$ of set-valued maps recursively by

$$W_N = \mathrm{cl}\, G_N,$$

$$W_{n-1} = \mathrm{cl}(G_{n-1} \cap \mathrm{ri}\, Y_{n-1}), \quad Y_{n-1} = \mathrm{conv}\, \mathcal{K}(W_n, \mathcal{F}_{n-1}), \quad 1 \le n \le N.$$

This sequence is well-defined and is adapted to the filtration. Indeed, suppose the map W_n is $\mathcal{F}_n$-measurable. If $W_n \neq \emptyset$ a.s., then the map $\mathrm{conv}\, \mathcal{K}(W_n, \mathcal{F}_{n-1})$ is $\mathcal{F}_{n-1}$-measurable (see [1, Proposition 4(a)] and [9, Proposition II.2.26]). Furthermore, the graphs of the maps G_{n-1}, $\mathrm{ri}\, Y_{n-1}$ are measurable with respect to the σ-algebra $\mathcal{F}_{n-1} \otimes \mathcal{B}$ [13, Lemma 1(c)]. Consequently, the map $G_{n-1} \cap \mathrm{ri}\, Y_{n-1}$ is $\mathcal{F}_{n-1}$-measurable [9, Corollary II.1.34]. Its closure W_{n-1} has the same property [9, Proposition II.1.8].

Provided $W_n = \emptyset$ on a set of positive measure, we have $W_{n-1} = \emptyset$ by the definition.

Theorem 1. *The following conditions are equivalent:*

(a) *there exist an adapted to the filtration* $(\mathcal{F}_n)_{n=0}^N$ *stochastic process* $x = (x_n)_{n=0}^N$ *and an equivalent to* $\mathbf{P}$ *probability measure* $\mathbf{Q}$ *such that* $x_n \in \mathcal{S}(G_n, \mathcal{F}_n)$, $n \geq 0$ *and* x *is a* $\mathbf{Q}$*-martingale;*

(b) $W_n \neq \emptyset$ *a.s.*, $n = 0, \dots, N-1$.

Denote by $\mathbf{E}(f|\mathcal{H})$ the generalized conditional expectation of the $\mathcal{F}$-measurable random variable f with respect to $\mathcal{H}$ (under the measure $\mathbf{P}$) [12, p.229], [5, p.117]. The proof of Theorem 1 is based on the following result.

Lemma 1. *Let* F *be an* $\mathcal{F}$*-measurable set-valued mapping with nonempty closed convex values. For any* $\mathcal{H}$*-measurable selector* ξ *of the map* $\mathrm{ri}(\mathrm{conv}\,\mathcal{K}(F, \mathcal{H}))$ *there exist an element* $\eta \in \mathcal{S}(\mathrm{ri}\, F, \mathcal{F})$ *and an* $\mathcal{F}$*-measurable random variable* $\gamma > 0$ *such that*

$$\xi = \mathbf{E}(\gamma\eta|\mathcal{H}), \quad \mathbf{E}(\gamma|\mathcal{H}) = 1 \ \text{a.s.} \tag{4}$$

Proof. Let $\{f_i\}_{i=1}^\infty$, $f_i \in \mathcal{S}(\mathrm{ri}\, F, \mathcal{F})$ be a Castaing representation for $\mathrm{ri}\, F$. Since $\mathrm{ri}\, F \in \mathcal{F} \otimes \mathcal{B}$ ([13, Lemma 1(c)]), such a representation exists (see [9, Proposition II.2.17]).

Evidently, $\{f_i\}_{i=1}^\infty$ is also a Castaing representation for F. Applying (3) we get

$$\xi \in \mathrm{ri}(\mathrm{conv}\,\mathcal{K}(F, \mathcal{H})) = \mathrm{ri}\left(\mathrm{conv}\left(\mathrm{cl}\left(\bigcup_{i=1}^\infty \mathcal{K}(f_i, \mathcal{H})\right)\right)\right) \ \text{a.s.}$$

Note that for any collection of sets $\{A_i\}_{i=1}^\infty$, $A_i \subset \mathbb{R}^d$ the following inclusion holds true

$$B_1 = \mathrm{ri}\left(\mathrm{conv}\left(\mathrm{cl}\left(\bigcup_{i=1}^\infty A_i\right)\right)\right) \subset \mathrm{conv}\left(\bigcup_{i=1}^\infty \mathrm{ri}\,(\mathrm{conv}\, A_i)\right) = B_2.$$

Indeed, suppose $x \notin B_2$. Then by the separation theorem there exist $p \in \mathbb{R}^d$, $j \in \mathbb{N}$, $\overline{y} \in A_j$ such that

$$\langle p, x\rangle \geq \langle p, y\rangle, \quad y \in A_i, \ i \in \mathbb{N};$$

$$\langle p, x\rangle > \langle p, \overline{y}\rangle.$$

Here $\langle \cdot, \cdot \rangle$ is the usual scalar product in $\mathbb{R}^d$. Obviously, $\langle p, x\rangle \geq \langle p, z\rangle$ for all $z \in \mathrm{cl}\, B_1$. Since $\overline{y} \in \mathrm{cl}\, B_1$, it follows that $\{x\}$ and $\mathrm{cl}\, B_1$ are properly separated. Therefore, $x \notin \mathrm{ri}(\mathrm{cl}\, B_1) = B_1$.

Putting $A_i = \mathcal{K}(f_i, \mathcal{H})$, we conclude that

$$\xi \in \mathrm{conv}\left(\bigcup_{i=1}^\infty \mathrm{ri}(\mathrm{conv}\,\mathcal{K}(f_i, \mathcal{H}))\right) \ \text{a.s.}$$

The results of the theory of measurable set-valued maps mentioned above, readily imply that ξ has an $\mathcal{H}$-measurable Caratheodory representation:

$$\xi = \sum_{k=1}^{d+1} \alpha_k \xi_k \ \text{a.s.}, \quad \xi_k \in \mathcal{S}\left(\bigcup_{i=1}^\infty \mathrm{ri}(\mathrm{conv}\,\mathcal{K}(f_i, \mathcal{H})), \mathcal{H}\right),$$

where $\mathcal{H}$-measurable functions α_k satisfy conditions (2).

Put $A_k^i = \{\omega : \xi_k \in \mathrm{ri}(\mathrm{conv}\,\mathcal{K}(f_i, \mathcal{H}))\}$ and consider the covering of Ω, consisting of the sets $A_1^{i_1} \cap \dots \cap A_{d+1}^{i_{d+1}}$, where the upper indexes run through all natural numbers. It is easy to show (see [1, Lemma 2]) that there exists an $\mathcal{H}$-measurable partition $\{D_j\}_{j\in J}$, $J \subset \mathbb{N}$ of Ω such that

$$\emptyset \neq D_j \subset A_1^{i_1} \cap \dots \cap A_{d+1}^{i_{d+1}}, \quad j \in J,$$

where the set $(i_1, \dots, i_{d+1})$ depends on j.

For almost all $\omega \in D_j$ we have

$$\xi_k \in \mathrm{ri}(\mathrm{conv}\,\mathcal{K}(f_{i_k(j)}, \mathcal{H})), \quad k = 1, \dots, d+1,$$

or, in other words, $0 \in \mathrm{ri}(\mathrm{conv}\,\mathcal{K}(\zeta_{kj}, \mathcal{H}))$ a.s., where $\zeta_{kj} = I_{D_j}(f_{i_k(j)} - \xi_k)$.

According to [4, Theorem 3] it follows that for any $k \in \{1, \dots, d+1\}$ and $j \in J$ there exists an equivalent to $\mathbf{P}$ probability measure $\mathbf{Q}_{kj}$ with a.s. bounded density $0 < \gamma_{kj} = d\mathbf{Q}_{kj}/d\mathbf{P}$ such that

$$I_{D_j}\xi_k = I_{D_j}\mathbf{E}_{\mathbf{Q}_{kj}}(f_{i_k(j)}|\mathcal{H}) = \frac{I_{D_j}}{\mathbf{E}(\gamma_{kj}|\mathcal{H})}\mathbf{E}(\gamma_{kj} f_{i_k(j)}|\mathcal{H}) \text{ a.s.}$$

In the last equality the generalized Bayes formula [5, Ch. V, §3a] is used.

Hence, we get the representation

$$\xi = \sum_{k=1}^{d+1} \alpha_k \xi_k = \mathbf{E}\left(\sum_{k=1}^{d+1} \frac{\alpha_k \gamma_{kj}}{\mathbf{E}(\gamma_{kj}|\mathcal{H})} f_{i_k(j)} \middle| \mathcal{H} \right) \text{ a.s. on } D_j.$$

Here we take into account that the equality $\mathbf{E}(gh|\mathcal{H}) = h\mathbf{E}(g|\mathcal{H})$ holds true if the function g is $\mathcal{F}$-measurable and $\mathbf{P}$-integrable, and the function h is $\mathcal{H}$-measurable (see the remark in [12, p. 236]).

Put $\beta_{kj} = \gamma_{kj}/\mathbf{E}(\gamma_{kj}|\mathcal{H})$ and introduce the functions

$$\gamma_j = \sum_{k=1}^{d+1} \alpha_k \beta_{kj}, \quad \eta_j = \sum_{k=1}^{d+1} \frac{\alpha_k \beta_{kj}}{\gamma_j} f_{i_k(j)}.$$

We have

$$\xi = \mathbf{E}(\gamma_j \eta_j|\mathcal{H}) \text{ a.s. on } D_j.$$

It remains to note that $\gamma_j > 0$, $\mathbf{E}(\gamma_j|\mathcal{H}) = 1$,

$$\eta_j \in \mathrm{conv}\{f_{i_1(j)}, \dots, f_{i_{d+1}(j)}\} \subset \mathrm{ri}\, F \text{ a.s. on } D_j,$$

and the functions

$$\gamma = \sum_{j\in J} I_{D_j}\gamma_j, \quad \eta = \sum_{j\in J} I_{D_j}\eta_j$$

satisfy conditions (4). The proof of Lemma 1 is complete.

Proof of Theorem 1. Assume that condition (b) is satisfied. Starting from an arbitrary selector $x_0 \in \mathcal{S}(\operatorname{ri} W_0, \mathcal{F}_0)$ let us construct adapted sequences $x_n \in \operatorname{ri} W_n$, $\gamma_n > 0$, meeting the conditions

$$x_{n-1} = \mathbf{E}(\gamma_n x_n | \mathcal{F}_n), \quad \mathbf{E}(\gamma_n | \mathcal{F}_{n-1}) = 1 \text{ a.s.}, \quad n = 1, \dots, N.$$

The existence of the selector x_0 is implied by already mentioned results [13, Lemma 1(c)], [9, Proposition II.2.17]. The existence of the above sequences follows from Lemma 1, since $x_{n-1} \in \mathcal{S}(\operatorname{ri} W_{n-1}, \mathcal{F}_{n-1})$ imply that $x_{n-1} \in \mathcal{S}(\operatorname{ri}(\operatorname{conv} \mathcal{K}(W_n, \mathcal{F}_{n-1})), \mathcal{F}_{n-1})$.

Consider the positive $\mathbf{P}$-martingale

$$(z_n)_{n=0}^N, \quad z_0 = 1, \quad z_n = \prod_{k=1}^n \gamma_k, \ n \geq 1$$

and the equivalent to $\mathbf{P}$ probability measure $\mathbf{Q}'$ with the density $d\mathbf{Q}'/d\mathbf{P} = z_N$. According to the generalized Bayes formula we have

$$x_{n-1} = \frac{1}{z_{n-1}} \mathbf{E}(x_n z_n | \mathcal{F}_{n-1}) = \mathbf{E}_{\mathbf{Q}'}(x_n | \mathcal{F}_{n-1}) \text{ a.s.}$$

Thus, the process x is a generalized (or, equivalently, a local) $\mathbf{Q}'$-martingale and it admits an equivalent martingale measure $\mathbf{Q}$ ([4, Theorem 3]).

As long as, moreover, $x_n \in \mathcal{S}(\operatorname{ri} W_n, \mathcal{F}_n) \subset \mathcal{S}(G_n, \mathcal{F}_n)$, condition (a) is verified.

Now assume that condition (a) is satisfied. Note that $x_N \in G_N \subset W_N$. Suppose the relations $x_j \in W_j$, $j \geq n$ are already established. Since

$$0 \in \operatorname{ri}(\operatorname{conv} \mathcal{K}(x_n - x_{n-1}, \mathcal{F}_{n-1})) \text{ a.s.}, \ n \geq 1$$

(see [4, Theorem 3]) and $\mathcal{K}(x_n, \mathcal{F}_{n-1}) \subset \mathcal{K}(W_n, \mathcal{F}_{n-1})$, it follows that

$$x_{n-1} \in G_{n-1} \cap \operatorname{ri}(\operatorname{conv} \mathcal{K}(x_n, \mathcal{F}_{n-1})) \subset G_{n-1} \cap \operatorname{ri} Y_{n-1} \subset W_{n-1} \text{ a.s.}$$

Particulary, $W_n \neq \emptyset$ a.s. for all n. The proof is complete.

In the paper [1] Theorem 1 was proved under one of the following additional assumptions: (i) the sets $G_n(\omega)$ are open; (ii) the set Ω is finite.

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